

A Study of the Differences and Similarities Between Mohand and Elzaki Transforms

Jack Adams¹, William Harris¹, Grace Jenkins², Ryan Parker³, and Lucas Wright^{*3}

¹ Department of Environmental Science, University of Queensland, Australia

² Department of Chemistry, University of Oxford, UK

³ Department of Mechanical Engineering, University of Manchester, UK

ABSTRACT

Many advanced problems, which appear in the field of engineering and sciences like heat conduction problems, mechanical oscillation problems, vibrating beams problems, electric circuit problems, population growth and radioactive decay problems, can be solved by integral transforms. In this paper, we present a comparative study of two integral transforms namely Mohand and Elzaki transforms and solve some systems of differential equations (Homogeneous & Non-Homogeneous) using both the transforms in application section. Results show that Mohand and Elzaki transforms are closely connected.

Keywords: Mohand transform, Elzaki transform, System of differential equations.

I. INTRODUCTION

The advanced problems of mathematics, physics, chemistry, social science, biology, radio physics, astronomy, nuclear science, electronics, chemical and mechanical engineering can be solved using integral transforms (Laplace transform [1, 7-11], Fourier transform [1], Aboodh transform [2], Hankel transform [1], Z-transform [1, 11], Wavelet transform [1], Elzaki transform [4], Mahgoub transform [5], Mohand transform [6], Sumudu transform [12], Mellin transform [1], Hermite transform [1], Kamal transform [3], etc.). Many real world problems, which are mathematically represented by differential equations, delay differential equations, partial differential equations, partial integro-differential equations, integral equations, integro-differential equations, solved by many scholars [13-28] using these transforms.

Aggarwal et al. [29] solved the problems of population growth and decay by applying Mohand transform. Aggarwal et al. [30] gave Mohand transform of Bessel's functions. Kumar et al. [31] applied Mohand transform and solved first kind linear Volterra integral equations. Kumar et al. [32] applied Mohand transform for solving the mechanics and electrical circuit problems. Aggarwal et al. [33] gave the solution of second kind linear Volterra integral equations using Mohand transform. Sathya and Rajeswari [34] used Mohand transform and solved linear partial integro-differential equations. The solution of linear Volterra integro-differential equations using Mohand transform was given by Kumar et al. [35].

Elzaki and Ezaki [36] used Elzaki transform and solved ordinary differential equation with variable coefficients. Elzaki and Ezaki [37] used Elzaki transform for solving partial differential equations. Shendkar and Jadhav [38] applied Elzaki transform for solving differential equations. Aggarwal et al. [39] used Elzaki transform for solving population growth and decay problems. Aggarwal et al. [40] applied Elzaki transform for solving linear Volterra integral equations of first kind. Aggarwal [41] defined Elzaki transform of Bessel's functions. A comparative study of Mohand and Laplace transforms was given by Aggarwal and Chaudhary [42].

In this paper, we concentrate mainly on the comparative study of Mohand and Elzaki transforms and we solve some systems of differential equations using these transforms.

II. DEFINITION OF MOHAND AND ELZAKI TRANSFORMS

2.1 Definition of Mohand transforms:

Mohand and Mahgoub [6] defined "Mohand transform" of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2 \int_0^{\infty} F(t)e^{-vt} dt = R(v), 0 < k_1 \leq v \leq k_2,$$

where the operator M is called the Mohand transform operator.

2.2 Definition of Elzaki transforms:

Elzaki [4] defined a new integral transform “Elzaki transform” of the function $F(t)$ for $t \geq 0$ as

$$E\{F(t)\} = v \int_0^\infty F(t) e^{-\frac{t}{v}} dt = T(v), \quad 0 < k_1 \leq v \leq k_2,$$

where the operator E is called the Elzaki transform operator.

If $F(t)$ is piecewise continuous and of exponential order then Mohand and Elzaki transforms of the function $F(t)$ for $t \geq 0$ exist. These two conditions are sufficient conditions for the existence of Mohand and Elzaki transforms of the function $F(t)$.

III. PROPERTIES OF MOHAND AND ELZAKI TRANSFORMS

In this section, we present some useful properties of Mohand and Elzaki transforms like the linearity property, change of scale property, first shifting theorem and convolution theorem.

3.1 Linearity property of Mohand and Elzaki transforms:

- a. **Linearity property of Mohand transforms [29-30, 33]:** If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$ is given by $[aR_1(v) + bR_2(v)]$, where a, b are arbitrary constants.
- b. **Linearity property of Elzaki transforms [39-41]:** If Elzaki transform of functions $F_1(t)$ and $F_2(t)$ are $T_1(v)$ and $T_2(v)$ respectively then Elzaki transform of $[aF_1(t) + bF_2(t)]$ is given by $[aT_1(v) + bT_2(v)]$, where a, b are arbitrary constants.

3.2 Change of scale property of Mohand and Elzaki transforms:

- a. **Change of scale property of Mohand transforms [30, 33]:** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $F(at)$ is given by $aR\left(\frac{v}{a}\right)$.
- b. **Change of scale property of Elzaki transforms [41]:** If Elzaki transform of function $F(t)$ is $T(v)$ then Elzaki transform of function $F(at)$ is given by $\frac{1}{a^2}T(av)$.

3.3 Shifting property of Mohand and Elzaki transforms:

- a. **Shifting property of Mohand transforms [33]:** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $e^{at}F(t)$ is given by $\frac{v^2}{(v-a)^2}R(v-a)$.
- b. **Shifting property of Elzaki transforms:** If Elzaki transform of function $F(t)$ is $T(v)$ then Elzaki transform of function $e^{at}F(t)$ is given by $(1-av)T\left(\frac{v}{(1-av)}\right)$.

Proof: By the definition of Elzaki transform, we have

$$\begin{aligned} E\{e^{at}F(t)\} &= v \int_0^\infty e^{at}F(t) e^{-\frac{t}{v}} dt = v \int_0^\infty F(t) e^{-\left[\frac{1}{v}-a\right]t} dt \\ &= v \int_0^\infty F(t) e^{-\frac{t}{v/(1-av)}} dt = (1-av) \left[\frac{v}{(1-av)} \int_0^\infty F(t) e^{-\frac{t}{v/(1-av)}} dt \right] = (1-av)T\left(\frac{v}{1-av}\right). \end{aligned}$$

3.4 Convolution theorem for Mohand and Elzaki transforms:

- a. **Convolution theorem for Mohand transforms [31, 33, 35]:** If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of their convolution $F_1(t) * F_2(t)$ is given by $M\{F_1(t) * F_2(t)\} = \frac{1}{v^2}M\{F_1(t)\}M\{F_2(t)\}$
 $\Rightarrow M\{F_1(t) * F_2(t)\} = \frac{1}{v^2}R_1(v)R_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$$
- b. **Convolution theorem for Elzaki transforms [40-41]:** If Elzaki transform of functions $F_1(t)$ and $F_2(t)$ are $T_1(v)$ and $T_2(v)$ respectively then Elzaki transform of their convolution $F_1(t) * F_2(t)$ is given by

$$E\{F_1(t) * F_2(t)\} = \frac{1}{v} E\{F_1(t)\} E\{F_2(t)\}$$

$$\Rightarrow E\{F_1(t) * F_2(t)\} = \frac{1}{v} T_1(v) T_2(v), \text{ where } F_1(t) * F_2(t) \text{ is defined by}$$

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x) F_2(x) dx = \int_0^t F_1(x) F_2(t-x) dx$$

IV. MOHAND AND ELZAKI TRANSFORMS OF THE DERIVATIVES OF THE FUNCTION $F(t)$

4.1 Mohand transforms of the derivatives of the function $F(t)$ [29-30]:

If $M\{F(t)\} = R(v)$ then

- $M\{F'(t)\} = vR(v) - v^2F(0)$
- $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
- $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \dots - v^2F^{(n-1)}(0)$

4.2 Elzaki transforms of the derivatives of the function $F(t)$ [39, 41]:

If $E\{F(t)\} = T(v)$ then

- $E\{F'(t)\} = \frac{1}{v}T(v) - vF(0)$
- $E\{F''(t)\} = \frac{1}{v^2}T(v) - F(0) - vF'(0)$
- $E\{F^{(n)}(t)\} = \frac{1}{v^n}T(v) - \frac{1}{v^{n-2}}F(0) - \frac{1}{v^{n-3}}F'(0) \dots - vF^{(n-1)}(0)$

V. MOHAND AND ELZAKI TRANSFORMS OF FREQUENTLY USED FUNCTIONS [29-33, 39-41]

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$	$E\{F(t)\} = T(v)$
1.	1	v	v^2
2.	t	1	v^3
3.	t^2	$\frac{2!}{v}$	$2! v^4$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$	$n! v^{n+2}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$	$\Gamma(n+1)v^{n+2}$
6.	e^{at}	$\frac{v^2}{v-a}$	$\frac{v^2}{1-av}$
7.	$\sin at$	$\frac{av^2}{(v^2+a^2)}$	$\frac{av^3}{1+a^2v^2}$
8.	$\cos at$	$\frac{v^3}{(v^2+a^2)}$	$\frac{v^2}{1+a^2v^2}$
9.	$\sinh at$	$\frac{av^2}{(v^2-a^2)}$	$\frac{av^3}{1-a^2v^2}$
10.	$\cosh at$	$\frac{v^3}{(v^2-a^2)}$	$\frac{v^2}{1-a^2v^2}$

VI. INVERSE MOHAND AND ELZAKI TRANSFORMS

6.1 Inverse Mohand transforms [29, 33, 42]: If $R(v)$ is the Mohand transform of $F(t)$ then $F(t)$ is called the inverse Mohand transform of $R(v)$ and in mathematical terms, it can be expressed as

$F(t) = M^{-1}\{R(v)\}$, where M^{-1} is an operator and it is called as inverse Mohand transform operator.

6.2 Inverse Elzaki transforms [39-41]: If $T(v)$ is the Elzaki transforms of $F(t)$ then $F(t)$ is called the inverse Elzaki transform of $T(v)$ and in mathematical terms, it can be expressed as $F(t) = E^{-1}\{T(v)\}$, where E^{-1} is an operator and it is called as inverse Elzaki transform operator.

VII. INVERSE MOHAND AND ELZAKI TRANSFORMS OF FREQUENTLY USED FUNCTIONS [29, 39-41]

Table: 2

S.N.	$R(v)$	$F(t) = M^{-1}\{R(v)\} = E^{-1}\{T(v)\}$	$T(v)$
1.	v	1	v^2
2.	1	t	v^3
3.	$\frac{1}{v}$	$\frac{t^2}{2}$	v^4
4.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{n!}, n \in N$	v^{n+2}
5.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{\Gamma(n+1)}, n > -1$	v^{n+2}
6.	$\frac{v^2}{v-a}$	e^{at}	$\frac{v^2}{1-av}$
7.	$\frac{v^2}{(v^2+a^2)}$	$\frac{\sin at}{a}$	$\frac{v^3}{1+a^2v^2}$
8.	$\frac{v^3}{(v^2+a^2)}$	$\cos at$	$\frac{v^2}{1+a^2v^2}$
9.	$\frac{v^2}{(v^2-a^2)}$	$\frac{\sinh at}{a}$	$\frac{v^3}{1-a^2v^2}$
10.	$\frac{v^3}{(v^2-a^2)}$	$\cosh at$	$\frac{v^2}{1-a^2v^2}$

VIII. APPLICATIONS OF MOHAND AND ELZAKI TRANSFORMS FOR SOLVING SYSTEM OF DIFFERENTIAL EQUATIONS

In this section some numerical applications are given to explain the procedure of solving the systems of differential equations (Homogeneous & Non-Homogeneous) using Mohand and Elzaki transforms.

8.1 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{d^2x}{dt^2} + 3x - 2y &= 0 \\ \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} - 3x + 5y &= 0 \end{aligned} \right\} \quad (1)$$

$$\text{with } x(0) = 0, y(0) = 0, x'(0) = 3, y'(0) = 2 \quad (2)$$

Solution using Mohand transforms:

Taking Mohand transform of system (1), we have

$$\left. \begin{aligned} M\left\{\frac{d^2x}{dt^2}\right\} + 3M\{x\} - 2M\{y\} &= 0 \\ M\left\{\frac{d^2x}{dt^2}\right\} + M\left\{\frac{d^2y}{dt^2}\right\} - 3M\{x\} + 5M\{y\} &= 0 \end{aligned} \right\} \quad (3)$$

Now using the property, Mohand transform of the derivatives of the function, in (3), we have

$$\left. \begin{aligned} v^2 M\{x\} - v^3 x(0) - v^2 x'(0) + 3M\{x\} - 2M\{y\} &= 0 \\ v^2 M\{x\} - v^3 x(0) - v^2 x'(0) + v^2 M\{y\} - v^3 y(0) - v^2 y'(0) - 3M\{x\} + 5M\{y\} &= 0 \end{aligned} \right\} \quad (4)$$

Using (2) in (4), we have

$$\left. \begin{aligned} (v^2 + 3)M\{x\} - 2M\{y\} &= 3v^2 \\ (v^2 - 3)M\{x\} + (v^2 + 5)M\{y\} &= 5v^2 \end{aligned} \right\} \quad (5)$$

Solving the system (5) for $M\{x\}$ and $M\{y\}$, we have

$$\left. \begin{aligned} M\{x\} &= \frac{11}{4} \left[\frac{v^2}{(v^2 + 1)} \right] + \frac{1}{4} \left[\frac{v^2}{(v^2 + 9)} \right] \\ M\{y\} &= \frac{11}{4} \left[\frac{v^2}{(v^2 + 1)} \right] - \frac{3}{4} \left[\frac{v^2}{(v^2 + 9)} \right] \end{aligned} \right\} \quad (6)$$

Now taking inverse Mohand transform of system (6), we have

$$\left. \begin{aligned} x &= \frac{11}{4} \sin t + \frac{1}{12} \sin 3t \\ y &= \frac{11}{4} \sin t - \frac{1}{4} \sin 3t \end{aligned} \right\} \quad (7)$$

which is the required solution of (1) with (2).

Solution using Elzaki transforms:

Taking Elzaki transform of system (1), we have

$$\left. \begin{aligned} E \left\{ \frac{d^2 x}{dt^2} \right\} + 3E\{x\} - 2E\{y\} &= 0 \\ E \left\{ \frac{d^2 x}{dt^2} \right\} + E \left\{ \frac{d^2 y}{dt^2} \right\} - 3E\{x\} + 5E\{y\} &= 0 \end{aligned} \right\} \quad (8)$$

Now using the property, Elzaki transform of the derivatives of the function, in (8), we have

$$\left. \begin{aligned} \frac{1}{v^2} E\{x\} - x(0) - vx'(0) + 3E\{x\} - 2E\{y\} &= 0 \\ \frac{1}{v^2} E\{x\} - x(0) - vx'(0) + \frac{1}{v^2} E\{y\} - y(0) - vy'(0) - 3E\{x\} + 5E\{y\} &= 0 \end{aligned} \right\} \quad (9)$$

Using (2) in (9), we have

$$\left. \begin{aligned} \left(\frac{1}{v^2} + 3 \right) E\{x\} - 2E\{y\} &= 3v \\ \left(\frac{1}{v^2} - 3 \right) E\{x\} + \left(\frac{1}{v^2} + 5 \right) E\{y\} &= 5v \end{aligned} \right\} \quad (10)$$

Solving the system (10) for $E\{x\}$ and $E\{y\}$, we have

$$\left. \begin{aligned} E\{x\} &= \frac{11}{4} \left[\frac{v^3}{1 + v^2} \right] + \frac{1}{4} \left[\frac{v^3}{1 + 9v^2} \right] \\ E\{y\} &= \frac{11}{4} \left[\frac{v^3}{1 + v^2} \right] - \frac{3}{4} \left[\frac{v^3}{1 + 9v^2} \right] \end{aligned} \right\} \quad (11)$$

Now taking inverse Elzaki transform of system (11), we have

$$\left. \begin{aligned} x &= \frac{11}{4} \sin t + \frac{1}{12} \sin 3t \\ y &= \frac{11}{4} \sin t - \frac{1}{4} \sin 3t \end{aligned} \right\} \quad (12)$$

which is the required solution of (1) with (2).

8.2 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{dx}{dt} + y &= 2\cos t \\ x + \frac{dy}{dt} &= 0 \end{aligned} \right\} \quad (13)$$

$$\text{with } x(0) = 0, y(0) = 1 \quad (14)$$

Solution using Mohand transforms:

Taking Mohand transform of system (13), we have

$$\left. \begin{aligned} M\left\{\frac{dx}{dt}\right\} + M\{y\} &= 2M\{\cos t\} \\ M\{x\} + M\left\{\frac{dy}{dt}\right\} &= 0 \end{aligned} \right\} \quad (15)$$

Now using the property, Mohand transform of the derivatives of the function, in (15), we have

$$\left. \begin{aligned} vM\{x\} - v^2x(0) + M\{y\} &= \frac{2v^3}{(v^2 + 1)} \\ M\{x\} + vM\{y\} - v^2y(0) &= 0 \end{aligned} \right\} \quad (16)$$

Using (14) in (16), we have

$$\left. \begin{aligned} vM\{x\} + M\{y\} &= \frac{2v^3}{(v^2 + 1)} \\ M\{x\} + vM\{y\} &= v^2 \end{aligned} \right\} \quad (17)$$

Solving the system (17) for $M\{x\}$ and $M\{y\}$, we have

$$\left. \begin{aligned} M\{x\} &= \left[\frac{v^2}{(v^2 + 1)} \right] \\ M\{y\} &= \left[\frac{v^3}{(v^2 + 1)} \right] \end{aligned} \right\} \quad (18)$$

Now taking inverse Mohand transform of system (18), we have

$$\left. \begin{aligned} x &= \sin t \\ y &= \cos t \end{aligned} \right\} \quad (19)$$

which is the required solution of (13) with (14).

Solution using Elzaki transforms:

Taking Elzaki transform of system (13), we have

$$\left. \begin{aligned} E\left\{\frac{dx}{dt}\right\} + E\{y\} &= 2E\{\cos t\} \\ E\{x\} + E\left\{\frac{dy}{dt}\right\} &= 0 \end{aligned} \right\} \quad (20)$$

Now using the property, Elzaki transform of the derivatives of the function, in (20), we have

$$\left. \begin{aligned} \frac{1}{v}E\{x\} - vx(0) + E\{y\} &= \frac{2v^2}{1 + v^2} \\ E\{x\} + \frac{1}{v}E\{y\} - vy(0) &= 0 \end{aligned} \right\} \quad (21)$$

Using (14) in (21), we have

$$\left. \begin{aligned} \frac{1}{v}E\{x\} + E\{y\} &= \frac{2v^2}{1+v^2} \\ E\{x\} + \frac{1}{v}E\{y\} &= v \end{aligned} \right\} \quad (22)$$

Solving the system (22) for $E\{x\}$ and $E\{y\}$, we have

$$\left. \begin{aligned} E\{x\} &= \left[\frac{v^3}{1+v^2} \right] \\ E\{y\} &= \left[\frac{v^2}{1+v^2} \right] \end{aligned} \right\} \quad (23)$$

Now taking inverse Elzaki transform of system (23), we have

$$\left. \begin{aligned} x &= \sin t \\ y &= \cos t \end{aligned} \right\} \quad (24)$$

which is the required solution of (13) with (14).

8.3 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{dz}{dt} + x &= \sin t \\ \frac{dx}{dt} - y &= e^t \\ \frac{dy}{dt} + z + x &= 1 \end{aligned} \right\} \quad (25)$$

$$\text{with } x(0) = 1, y(0) = 1, z(0) = 0 \quad (26)$$

Solution using Mohand transforms:

Taking Mohand transform of system (25), we have

$$\left. \begin{aligned} M\left\{\frac{dz}{dt}\right\} + M\{x\} &= M\{\sin t\} \\ M\left\{\frac{dx}{dt}\right\} - M\{y\} &= M\{e^t\} \\ M\left\{\frac{dy}{dt}\right\} + M\{z\} + M\{x\} &= M\{1\} \end{aligned} \right\} \quad (27)$$

Now using the property, Mohand transform of the derivatives of the function, in (27), we have

$$\left. \begin{aligned} vM\{z\} - v^2z(0) + M\{x\} &= \left[\frac{v^2}{(v^2+1)} \right] \\ vM\{x\} - v^2x(0) - M\{y\} &= \left[\frac{v^2}{v-1} \right] \\ vM\{y\} - v^2y(0) + M\{z\} + M\{x\} &= v \end{aligned} \right\} \quad (28)$$

Using (26) in (28), we have

$$\left. \begin{aligned} vM\{z\} + M\{x\} &= \left[\frac{v^2}{(v^2+1)} \right] \\ vM\{x\} - M\{y\} &= \left[\frac{v^3}{v-1} \right] \\ vM\{y\} + M\{z\} + M\{x\} &= v + v^2 \end{aligned} \right\} \quad (29)$$

Solving the system (29) for $M\{x\}$, $M\{y\}$ and $M\{z\}$, we have

$$\left. \begin{aligned} M\{x\} &= \left[\frac{v^2}{v-1} \right] + \left[\frac{v^2}{(v^2+1)} \right] \\ M\{y\} &= \left[\frac{v^3}{(v^2+1)} \right] \\ M\{z\} &= v - \left[\frac{v^2}{v-1} \right] \end{aligned} \right\} \quad (30)$$

Now taking inverse Mohand transform of system (30), we have

$$\left. \begin{aligned} x &= e^t + \sin t \\ y &= \cos t \\ z &= 1 - e^t \end{aligned} \right\} \quad (31)$$

which is the required solution of (25) with (26).

Solution using Elzaki transforms:

Taking Elzaki transform of system (25), we have

$$\left. \begin{aligned} E\left\{\frac{dz}{dt}\right\} + E\{x\} &= E\{\sin t\} \\ E\left\{\frac{dx}{dt}\right\} - E\{y\} &= E\{e^t\} \\ E\left\{\frac{dy}{dt}\right\} + E\{z\} + E\{x\} &= E\{1\} \end{aligned} \right\} \quad (32)$$

Now using the property, Elzaki transform of the derivatives of the function, in (32), we have

$$\left. \begin{aligned} \frac{1}{v}E\{z\} - vz(0) + E\{x\} &= \left[\frac{v^3}{1+v^2} \right] \\ \frac{1}{v}E\{x\} - vx(0) - E\{y\} &= \left[\frac{v^2}{1-v} \right] \\ \frac{1}{v}E\{y\} - vy(0) + E\{z\} + E\{x\} &= v^2 \end{aligned} \right\} \quad (33)$$

Using (26) in (33), we have

$$\left. \begin{aligned} \frac{1}{v}E\{z\} + E\{x\} &= \left[\frac{v^3}{(v^2+1)} \right] \\ \frac{1}{v}E\{x\} - E\{y\} &= \left[\frac{v}{1-v} \right] \\ \frac{1}{v}E\{y\} + E\{z\} + E\{x\} &= v^2 + v \end{aligned} \right\} \quad (34)$$

Solving the system (34) for $E\{x\}$, $E\{y\}$ and $E\{z\}$, we have

$$\left. \begin{aligned} E\{x\} &= \left[\frac{v^2}{1-v} \right] + \left[\frac{v^3}{1+v^2} \right] \\ E\{y\} &= \left[\frac{v^2}{1+v^2} \right] \\ E\{z\} &= v^2 - \left[\frac{v^2}{1-v} \right] \end{aligned} \right\} \quad (35)$$

Now taking inverse Elzaki transform of system (35), we have

$$\left. \begin{aligned} x &= e^t + \sin t \\ y &= \cos t \\ z &= 1 - e^t \end{aligned} \right\} \quad (36)$$

which is the required solution of (25) with (26).

IX. CONCLUSIONS

In this paper, we have successfully discussed the comparative study of Mohand and Elzaki transforms. In application section, we solve systems of differential equations (Homogeneous & Non-Homogeneous) comparatively using both the transforms. The numerical applications which are given in application section show that both the transforms (Mohand and Elzaki transforms) are closely connected to each other.

REFERENCES

1. Lokenath Debnath and Bhatta, D., *Integral transforms and their applications*, Second edition, Chapman & Hall/CRC, 2006.
2. Aboodh, K.S., *The new integral transform "Aboodh Transform"*, *Global Journal of Pure and Applied Mathematics*, 9(1), 35-43, 2013.
3. Abdelilah, K. and Hassan, S., *The new integral transform "Kamal Transform"*, *Advances in Theoretical and Applied Mathematics*, 11(4), 451-458, 2016.
4. Elzaki, T.M., *The new integral transform "Elzaki Transform"*, *Global Journal of Pure and Applied Mathematics*, 1, 57-64, 2011.
5. Mahgoub, M.A.M., *The new integral transform "Mahgoub Transform"*, *Advances in Theoretical and Applied Mathematics*, 11(4), 391-398, 2016.
6. Mohand, M. and Mahgoub, A., *The new integral transform "Mohand Transform"*, *Advances in Theoretical and Applied Mathematics*, 12(2), 113 – 120, 2017.
7. Raisinghania, M.D., *Advanced differential equations*, S. Chand & Co. Ltd, 2015.
8. Jeffrey, A., *Advanced engineering mathematics*, Harcourt Academic Press, 2002.
9. Dass, H.K., *Advanced engineering mathematics*, S. Chand & Co. Ltd, 2007.
10. Greenberg, M.D., *Advanced engineering mathematics*, Prentice Hall, 1998.
11. Stroud, K.A. and Booth, D.J., *Engineering mathematics*, Industrial Press, Inc., 2001.
12. Watugula, G.K., *Sumudu transform: A new integral transform to solve differential equations and control engineering problems*, *International Journal of Mathematical Education in Science and Technology*, 24(1), 35-43, 1993.
13. Aggarwal, S., Sharma, N., Chauhan, R., Gupta, A.R. and Khandelwal, A., *A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients*, *Journal of Computer and Mathematical Sciences*, 9(6), 520-525, 2018.
14. Aggarwal, S., Chauhan, R. and Sharma, N., *A new application of Mahgoub transform for solving linear Volterra integral equations*, *Asian Resonance*, 7(2), 46-48, 2018.
15. Aggarwal, S., Sharma, N. and Chauhan, R., *Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform*, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(5), 173-176, 2018.
16. Aggarwal, S., Pandey, M., Asthana, N., Singh, D.P. and Kumar, A., *Application of Mahgoub transform for solving population growth and decay problems*, *Journal of Computer and Mathematical Sciences*, 9(10), 1490-1496, 2018.
17. Aggarwal, S., Gupta, A.R., Asthana, N. and Singh, D.P., *Application of Kamal transform for solving population growth and decay problems*, *Global Journal of Engineering Science and Researches*, 5(9), 254-260, 2018.
18. Abdelilah, K. and Hassan, S., *The use of Kamal transform for solving partial differential equations*, *Advances in Theoretical and Applied Mathematics*, 12(1), 7-13, 2017.
19. Aggarwal, S., Chauhan, R. and Sharma, N., *A new application of Kamal transform for solving linear Volterra integral equations*, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(4), 138-140, 2018.
20. Gupta, A.R., Aggarwal, S. and Agrawal, D., *Solution of linear partial integro-differential equations using Kamal transform*, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(7), 88-91, 2018.
21. Aggarwal, S., Sharma, N. and Chauhan, R., *Application of Kamal transform for solving linear Volterra integral equations of first kind*, *International Journal of Research in Advent Technology*, 6(8), 2081-2088, 2018.

22. Aboodh, K.S., *Application of new transform "Aboodh Transform" to partial differential equations*, *Global Journal of Pure and Applied Mathematics*, 10(2), 249-254, 2014.
23. Aboodh, K.S., Farah, R.A., Almardy, I.A. and Osman, A.K., *Solving delay differential equations by Aboodh transformation method*, *International Journal of Applied Mathematics & Statistical Sciences*, 7(2), 55-64, 2018.
24. Aboodh, K.S., Farah, R.A., Almardy, I.A. and Almostafa, F.A., *Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods*, *Global Journal of Pure and Applied Mathematics*, 13(8), 4347-4360, 2016.
25. Aggarwal, S., Sharma, N. and Chauhan, R., *Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind*, *International Journal of Research in Advent Technology*, 6(6), 1186-1190, 2018.
26. Aggarwal, S., Sharma, N. and Chauhan, R., *A new application of Aboodh transform for solving linear Volterra integral equations*, *Asian Resonance*, 7(3), 156-158, 2018.
27. Mohand, D., Aboodh, K.S. and Abdelbagy, A., *On the solution of ordinary differential equation with variable coefficients using Aboodh transform*, *Advances in Theoretical and Applied Mathematics*, 11(4), 383-389, 2016.
28. Aggarwal, S., Sharma, N. and Chauhan, R., *Solution of population growth and decay problems by using Mohand transform*, *International Journal of Research in Advent Technology*, 6(11), 3277-3282, 2018.
29. Aggarwal, S., Chauhan, R. and Sharma, N., *Mohand transform of Bessel's functions*, *International Journal of Research in Advent Technology*, 6(11), 3034-3038, 2018.
30. Kumar, P.S., Saranya, C., Gnanavel, M.G. and Viswanathan, A., *Applications of Mohand transform for solving linear Volterra integral equations of first kind*, *International Journal of Research in Advent Technology*, 6(10), 2786-2789, 2018.
31. Kumar, P.S., Gomathi, P., Gowri, S. and Viswanathan, A., *Applications of Mohand transform to mechanics and electrical circuit problems*, *International Journal of Research in Advent Technology*, 6(10), 2838-2840, 2018.
32. Aggarwal, S., Sharma, N. and Chauhan, R., *Solution of linear Volterra integral equations of second kind using Mohand transform*, *International Journal of Research in Advent Technology*, 6(11), 3098-3102, 2018.
33. Sathya, S. and Rajeswari, I., *Applications of Mohand transform for solving linear partial integro-differential equations*, *International Journal of Research in Advent Technology*, 6(10), 2841-2843, 2018.
34. Kumar, P.S., Gnanavel, M.G. and Viswanathan, A., *Application of Mohand transform for solving linear Volterra integro-differential equations*, *International Journal of Research in Advent Technology*, 6(10), 2554-2556, 2018.
35. Elzaki, T.M. and Ezaki, S.M., *On the Elzaki transform and ordinary differential equation with variable coefficients*, *Advances in Theoretical and Applied Mathematics*, 6(1), 41-46, 2011.
36. Elzaki, T.M. and Ezaki, S.M., *Applications of new transform "Elzaki transform" to partial differential equations*, *Global Journal of Pure and Applied Mathematics*, 7(1), 65-70, 2011.
37. Shendkar, A.M. and Jadhav, P.V., *Elzaki transform: A solution of differential equations*, *International Journal of Science, Engineering and Technology Research*, 4(4), 1006-1008, 2015.
38. Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R., *Application of Elzaki transform for solving population growth and decay problems*, *Journal of Emerging Technologies and Innovative Research*, 5(9), 281-284, 2018.
39. Aggarwal, S., Chauhan, R. and Sharma, N., *Application of Elzaki transform for solving linear Volterra integral equations of first kind*, *International Journal of Research in Advent Technology*, 6(12), 3687-3692, 2018.
40. Aggarwal, S., *Elzaki transform of Bessel's functions*, *Global Journal of Engineering Science and Researches*, 5(8), 45-51, 2018.
41. Aggarwal, S. and Chaudhary, R., *A comparative study of Mohand and Laplace transforms*, *Journal of Emerging Technologies and Innovative Research*, 6(2), 230-240, 2019.