

A Comparative Analysis of Gopal and Rayan Transforms

Rahul Mehta^{*1}, Priya Verma², Sneha Kapoor³ & Dr. Akshita Chauhan⁴

^{*1}Assistant Professor, Department of Mathematics, National P.G. College, Barhalganj,
Gorakhpur-273402, U.P., India

²Assistant Professor, Department of Mathematics, Noida Institute of Engineering & Technology, Greater
Noida-201306, U.P., India

³Assistant Professor, Department of Applied Science & Humanities, I.T.S. Engineering College, Greater
Noida-201308, U.P., India

⁴Professor, Department of Mathematics, Noida Institute of Engineering & Technology, Greater Noida-
201306, U.P., India

ABSTRACT

Mohand and Kamal transforms are very useful integral transforms for solving many advanced problems of engineering and sciences like heat conduction problems, vibrating beams problems, population growth and decay problems, electric circuit problems etc. In this article, we present a comparative study of two integral transforms namely Mohand and Kamal transforms. We solve some systems of differential equations using both the transforms in application section. Results show that Mohand and Kamal transforms are closely connected.

Keywords: Mohand transform, Kamal transform, System of differential equations

AMS SUBJECT CLASSIFICATION 2010: 44A05, 34A12, 44A35.

I. INTRODUCTION

In modern time, the advanced problems of mathematics, physics, chemistry, social science, biology, radio physics, astronomy, nuclear science, electronics, chemical and mechanical engineering can be solved using integral transforms (Laplace transform [1, 7-11], Fourier transform [1], Aboodh transform [2], Hankel transform [1], Z-transform [1, 11], Wavelet transform [1], Elzaki transform [4], Mahgoub transform [5], Mohand transform [6], Sumudu transform [12], Mellin transform [1], Hermite transform [1], Kamal transform [3], etc.).

Many scholars [13-28] applied these transforms and solve the problems of real world which are mathematically represented by differential equations, delay differential equations, partial differential equations, partial integro-differential equations, integral equations, integro-differential equations. Aggarwal et al. [29] applied Mohand transform and solve the problems of population growth and decay. Aggarwal et al. [30] defined Mohand transform of Bessel's functions. Kumar et al. [31] used Mohand transform and solved linear Volterra integral equations of first kind.

Kumar et al. [32] gave the applications of Mohand transform for solving the mechanics and electrical circuit problems. Aggarwal et al. [33] used Mohand transform for solving linear Volterra integral equations of second kind. Sathya and Rajeswari [34] applied Mohand transform for solving linear partial integro-differential equations. Application of Mohand transform for solving linear Volterra integro-differential equations was given by Kumar et al. [35]. Aggarwal [36] defined Kamal transform of Bessel's functions. Abdelilah and Hassan [37] used Kamal transform for solving partial differential equations. Aggarwal et al. [38] gave a new application of Kamal transform for solving linear Volterra integral equations. Solution of linear partial integro-differential equations using Kamal transform was given by Gupta et al. [39]. Aggarwal et al. [40] applied Kamal transform for solving linear Volterra integral equations of first kind. Application of Kamal transform for solving population growth and decay problems was given by Aggarwal et al. [41]. Aggarwal and Chaudhary [42] gave a comparative study of Mohand and Laplace transforms.

In this paper, we concentrate mainly on the comparative study of Mohand and Kamal transforms and we solve some systems of differential equations using these transforms.

II. DEFINITION OF MOHAND AND KAMAL TRANSFORMS

2.1 Definition of Mohand transforms:

In year 2017, Mohand and Mahgoub [6] defined “Mohand transform” of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2 \int_0^\infty F(t) e^{-vt} dt = R(v), k_1 \leq v \leq k_2,$$

where the operator M is called the Mohand transform operator.

2.2 Definition of Kamal transforms:

In year 2016, Abdelilah and Hassan [3] defined “Kamal transform” of the function $F(t)$ for $t \geq 0$ as

$$K\{F(t)\} = \int_0^\infty F(t) e^{-\frac{t}{v}} dt = G(v), k_1 \leq v \leq k_2,$$

where the operator K is called the Kamal transform operator.

The Mohand and Kamal transforms of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mohand and Kamal transforms of the function $F(t)$.

III. PROPERTIES OF MOHAND AND KAMAL TRANSFORMS

In this section, we present the linearity property, change of scale property, first shifting theorem, convolution theorem of Mohand and Kamal transforms.

3.1 Linearity property of Mohand and Kamal transforms:

- Linearity property of Mohand transforms [29-30, 33]:** If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$ is given by $[aR_1(v) + bR_2(v)]$, where a, b are arbitrary constants.
- Linearity property of Kamal transforms [36, 41]:** If Kamal transform of functions $F_1(t)$ and $F_2(t)$ are $G_1(v)$ and $G_2(v)$ respectively then Kamal transform of $[aF_1(t) + bF_2(t)]$ is given by $[aG_1(v) + bG_2(v)]$, where a, b are arbitrary constants.

3.2 Change of scale property of Mohand and Kamal transforms:

- Change of scale property of Mohand transforms [30, 33]:** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $F(at)$ is given by $aR\left(\frac{v}{a}\right)$.
- Change of scale property of Kamal transforms:** If Kamal transform of function $F(t)$ is $G(v)$ then Kamal transform of function $F(at)$ is given by $\frac{1}{a}G(av)$.

Proof: By the definition of Kamal transform, we have

$$K\{F(at)\} = \int_0^\infty F(at) e^{-\frac{t}{v}} dt \quad (1)$$

Put $at = p \Rightarrow adt = dp$ in equation(1), we have

$$\begin{aligned} K\{F(at)\} &= \frac{1}{a} \int_0^\infty F(p) e^{-\frac{p}{av}} dp \\ \Rightarrow K\{F(at)\} &= \frac{1}{a} G(av) \end{aligned}$$

3.3 Shifting property of Mohand and Kamal transforms:

- Shifting property of Mohand transforms [33]:** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $e^{at}F(t)$ is given by $\frac{v^2}{(v-a)^2} R(v-a)$.
- Shifting property of Kamal transforms:** If Kamal transform of function $F(t)$ is $G(v)$ then Kamal transform of function $e^{at}F(t)$ is given by $G\left(\frac{v}{1-av}\right)$.

Proof: By the definition of Kamal transform, we have

$$K\{e^{at}F(t)\} = \int_0^\infty e^{at}F(t) e^{-\frac{t}{v}} dt = \int_0^\infty F(t) e^{-\left[\frac{1}{v}-a\right]t} dt$$

$$= \int_0^{\infty} F(t) e^{-\left[\frac{t}{v/(1-av)}\right]} dt = G\left(\frac{v}{1-av}\right)$$

3.4 Convolution theorem for Mohand and Kamal transforms:

a. Convolution theorem for Mohand transforms [31, 33, 35]: If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of their convolution $F_1(t) * F_2(t)$ is given by

$$M\{F_1(t) * F_2(t)\} = \frac{1}{v^2} M\{F_1(t)\} M\{F_2(t)\}$$

$\Rightarrow M\{F_1(t) * F_2(t)\} = \frac{1}{v^2} R_1(v) R_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x) F_2(x) dx = \int_0^t F_1(x) F_2(t-x) dx$$

b. Convolution theorem for Kamal transforms [36, 38, 40]: If Kamal transform of functions $F_1(t)$ and $F_2(t)$ are $G_1(v)$ and $G_2(v)$ respectively then Kamal transform of their convolution $F_1(t) * F_2(t)$ is given by

$$K\{F_1(t) * F_2(t)\} = K\{F_1(t)\} K\{F_2(t)\}$$

$\Rightarrow K\{F_1(t) * F_2(t)\} = G_1(v) G_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x) F_2(x) dx = \int_0^t F_1(x) F_2(t-x) dx$$

IV. MOHAND AND KAMAL TRANSFORMS OF THE DERIVATIVES OF THE FUNCTION $F(t)$

4.1 Mohand transforms of the derivatives of the function $F(t)$ [29-30]:

If $M\{F(t)\} = R(v)$ then

- a) $M\{F'(t)\} = vR(v) - v^2 F(0)$
- b) $M\{F''(t)\} = v^2 R(v) - v^3 F(0) - v^2 F'(0)$
- c) $M\{F^{(n)}(t)\} = v^n R(v) - v^{n+1} F(0) - v^n F'(0) - \dots - v^2 F^{(n-1)}(0)$

4.2 Kamal transforms of the derivatives of the function $F(t)$ [3, 36, 39, 41]:

If $K\{F(t)\} = G(v)$ then

- a) $K\{F'(t)\} = \frac{1}{v} G(v) - F(0)$
- b) $K\{F''(t)\} = \frac{1}{v^2} G(v) - \frac{1}{v} F(0) - F'(0)$
- c) $K\{F^{(n)}(t)\} = \frac{1}{v^n} G(v) - \frac{1}{v^{n-1}} F(0) - \frac{1}{v^{n-2}} F'(0) - \dots - F^{(n-1)}(0)$

V. MOHAND AND KAMAL TRANSFORMS OF FREQUENTLY USED FUNCTIONS [29-33, 3, 36, 38, 40-42]

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$	$K\{F(t)\} = G(v)$
1.	1	v	v
2.	t	1	v^2
3.	t^2	$\frac{2!}{v}$	$2! v^3$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$	$n! v^{n+1}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$	$\Gamma(n+1) v^{n+1}$
6.	e^{at}	$\frac{v^2}{v-a}$	$\frac{v}{1-av}$
7.	$\sin at$	$\frac{av^2}{(v^2+a^2)}$	$\frac{av^2}{1+a^2 v^2}$

8.	$\cos at$	$\frac{v^3}{(v^2 + a^2)}$	$\frac{v}{1 + a^2 v^2}$
9.	$\sin hat$	$\frac{av^2}{(v^2 - a^2)}$	$\frac{av^2}{1 - a^2 v^2}$
10.	$\cosh at$	$\frac{v^3}{(v^2 - a^2)}$	$\frac{v}{1 - a^2 v^2}$
11.	$J_0(t)$	$\frac{v^2}{\sqrt{(1 + v^2)}}$	$\frac{v}{\sqrt{(1 + v^2)}}$
12.	$J_1(t)$	$v^2 - \frac{v^3}{\sqrt{(1 + v^2)}}$	$1 - \frac{1}{\sqrt{(1 + v^2)}}$

VI. INVERSE MOHAND AND KAMAL TRANSFORMS:

6.1 Inverse Mohand transforms [29, 33, 42]: If $R(v)$ is the Mohand transform of $F(t)$ then $F(t)$ is called the inverse Mohand transform of $R(v)$ and in mathematical terms, it can be expressed as $F(t) = M^{-1}\{R(v)\}$, where M^{-1} is an operator and it is called as inverse Mohand transform operator.

6.2 Inverse Kamal transforms [36, 38, 41]: If $G(v)$ is the Kamal transforms of $F(t)$ then $F(t)$ is called the inverse Kamal transform of $G(v)$ and in mathematical terms, it can be expressed as $F(t) = K^{-1}\{G(v)\}$, where K^{-1} is an operator and it is called as inverse Kamal transform operator.

VII. INVERSE MOHAND AND KAMAL TRANSFORMS OF FREQUENTLY USED FUNCTIONS [29, 36, 38, 41-42]

Table: 2

S.N.	$R(v)$	$F(t) = M^{-1}\{R(v)\} = K^{-1}\{G(v)\}$	$G(v)$
1.	v	1	v
2.	1	t	v^2
3.	$\frac{1}{v}$	$\frac{t^2}{2}$	v^3
4.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{n!}, n \in N$	v^{n+1}
5.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{\Gamma(n+1)}, n > -1$	v^{n+1}
6.	$\frac{v^2}{v-a}$	e^{at}	$\frac{v}{1-av}$
7.	$\frac{v^2}{(v^2+a^2)}$	$\frac{\sin at}{a}$	$\frac{v^2}{1+a^2 v^2}$
8.	$\frac{v^3}{(v^2+a^2)}$	$\cos at$	$\frac{v}{1+a^2 v^2}$
9.	$\frac{v^2}{(v^2-a^2)}$	$\frac{\sin hat}{a}$	$\frac{v^2}{1-a^2 v^2}$
10.	$\frac{v^3}{(v^2-a^2)}$	$\cosh at$	$\frac{v}{1-a^2 v^2}$
11.	$\frac{v^2}{\sqrt{(1+v^2)}}$	$J_0(t)$	$\frac{v}{\sqrt{(1+v^2)}}$

12.	$v^2 - \frac{v^3}{\sqrt{(1+v^2)}}$	$J_1(t)$	$1 - \frac{1}{\sqrt{(1+v^2)}}$
-----	------------------------------------	----------	--------------------------------

VIII. APPLICATIONS OF MOHAND AND KAMAL TRANSFORMS FOR SOLVING SYSTEM OF DIFFERENTIAL EQUATIONS

In this section some numerical applications are given to explain the procedure of solving the systems of differential equations (Homogeneous & Non-Homogeneous) using Mohand and Kamal transforms.

8.1 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{d^2x}{dt^2} + 3x - 2y &= 0 \\ \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} - 3x + 5y &= 0 \end{aligned} \right\} \quad (2)$$

$$\text{with } x(0) = 0, y(0) = 0, x'(0) = 3, y'(0) = 2 \quad (3)$$

Solution using Mohand transforms:

Taking Mohand transform of system (2), we have

$$\left. \begin{aligned} M\left\{\frac{d^2x}{dt^2}\right\} + 3M\{x\} - 2M\{y\} &= 0 \\ M\left\{\frac{d^2x}{dt^2}\right\} + M\left\{\frac{d^2y}{dt^2}\right\} - 3M\{x\} + 5M\{y\} &= 0 \end{aligned} \right\} \quad (4)$$

Now using the property, Mohand transform of the derivatives of the function, in (4), we have

$$\left. \begin{aligned} v^2 M\{x\} - v^3 x(0) - v^2 x'(0) + 3M\{x\} - 2M\{y\} &= 0 \\ v^2 M\{x\} - v^3 x(0) - v^2 x'(0) + v^2 M\{y\} - v^3 y(0) - v^2 y'(0) - 3M\{x\} + 5M\{y\} &= 0 \end{aligned} \right\} \quad (5)$$

Using (3) in (5), we have

$$\left. \begin{aligned} (v^2 + 3)M\{x\} - 2M\{y\} &= 3v^2 \\ (v^2 - 3)M\{x\} + (v^2 + 5)M\{y\} &= 5v^2 \end{aligned} \right\} \quad (6)$$

Solving the system (6) for $M\{x\}$ and $M\{y\}$, we have

$$\left. \begin{aligned} M\{x\} &= \frac{11}{4} \left[\frac{v^2}{(v^2 + 1)} \right] + \frac{1}{4} \left[\frac{v^2}{(v^2 + 9)} \right] \\ M\{y\} &= \frac{11}{4} \left[\frac{v^2}{(v^2 + 1)} \right] - \frac{3}{4} \left[\frac{v^2}{(v^2 + 9)} \right] \end{aligned} \right\} \quad (7)$$

Now taking inverse Mohand transform of system (7), we have

$$\left. \begin{aligned} x &= \frac{11}{4} \sin t + \frac{1}{12} \sin 3t \\ y &= \frac{11}{4} \sin t - \frac{1}{4} \sin 3t \end{aligned} \right\} \quad (8)$$

which is the required solution of (2) with (3).

Solution using Kamal transforms:

Taking Kamal transform of system (2), we have

$$\left. \begin{aligned} K\left\{\frac{d^2x}{dt^2}\right\} + 3K\{x\} - 2K\{y\} &= 0 \\ K\left\{\frac{d^2x}{dt^2}\right\} + K\left\{\frac{d^2y}{dt^2}\right\} - 3K\{x\} + 5K\{y\} &= 0 \end{aligned} \right\} \quad (9)$$

Now using the property, Kamal transform of the derivatives of the function, in (9), we have

$$\left. \begin{aligned} \frac{1}{v^2} K\{x\} - \frac{1}{v} x(0) - x'(0) + 3K\{x\} - 2K\{y\} &= 0 \\ \frac{1}{v^2} K\{x\} - \frac{1}{v} x(0) - x'(0) + \frac{1}{v^2} K\{y\} - \frac{1}{v} y(0) - y'(0) - 3K\{x\} + 5K\{y\} &= 0 \end{aligned} \right\} \quad (10)$$

Using (3) in (10), we have

$$\left. \begin{aligned} \left(\frac{1}{v^2} + 3\right) K\{x\} - 2K\{y\} &= 3 \\ \left(\frac{1}{v^2} - 3\right) K\{x\} + \left(\frac{1}{v^2} + 5\right) K\{y\} &= 5 \end{aligned} \right\} \quad (11)$$

Solving the system (11) for $K\{x\}$ and $K\{y\}$, we have

$$\left. \begin{aligned} K\{x\} &= \frac{11}{4} \left[\frac{v^2}{1+v^2} \right] + \frac{1}{4} \left[\frac{v^2}{1+9v^2} \right] \\ K\{y\} &= \frac{11}{4} \left[\frac{v^2}{1+v^2} \right] - \frac{3}{4} \left[\frac{v^2}{1+9v^2} \right] \end{aligned} \right\} \quad (12)$$

Now taking inverse Kamal transform of system (12), we have

$$\left. \begin{aligned} x &= \frac{11}{4} \sin t + \frac{1}{12} \sin 3t \\ y &= \frac{11}{4} \sin t - \frac{1}{4} \sin 3t \end{aligned} \right\} \quad (13)$$

which is the required solution of (2) with (3).

8.2 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{dx}{dt} + y &= 2 \cos t \\ x + \frac{dy}{dt} &= 0 \end{aligned} \right\} \quad (14)$$

$$\text{with } x(0) = 0, y(0) = 1 \quad (15)$$

Solution using Mohand transforms:

Taking Mohand transform of system (14), we have

$$\left. \begin{aligned} M\left\{\frac{dx}{dt}\right\} + M\{y\} &= 2M\{\cos t\} \\ M\{x\} + M\left\{\frac{dy}{dt}\right\} &= 0 \end{aligned} \right\} \quad (16)$$

Now using the property, Mohand transform of the derivatives of the function, in (16), we have

$$\left. \begin{aligned} vM\{x\} - v^2x(0) + M\{y\} &= \frac{2v^3}{(v^2 + 1)} \\ M\{x\} + vM\{y\} - v^2y(0) &= 0 \end{aligned} \right\} \quad (17)$$

Using (15) in (17), we have

$$\left. \begin{aligned} vM\{x\} + M\{y\} &= \frac{2v^3}{(v^2 + 1)} \\ M\{x\} + vM\{y\} &= v^2 \end{aligned} \right\} \quad (18)$$

Solving the system (18) for $M\{x\}$ and $M\{y\}$, we have

$$\left. \begin{aligned} M\{x\} &= \left[\frac{v^2}{(v^2 + 1)} \right] \\ M\{y\} &= \left[\frac{v^3}{(v^2 + 1)} \right] \end{aligned} \right\} \quad (19)$$

Now taking inverse Mohand transform of system (19), we have

$$\left. \begin{aligned} x &= \sin t \\ y &= \cos t \end{aligned} \right\} \quad (20)$$

which is the required solution of (14) with (15).

Solution using Kamal transforms:

Taking Kamal transform of system (14), we have

$$\left. \begin{aligned} K\left\{\frac{dx}{dt}\right\} + K\{y\} &= 2K\{\cos t\} \\ K\{x\} + K\left\{\frac{dy}{dt}\right\} &= 0 \end{aligned} \right\} \quad (21)$$

Now using the property, Kamal transform of the derivatives of the function, in (21), we have

$$\left. \begin{aligned} \frac{1}{v}K\{x\} - x(0) + K\{y\} &= \frac{2v}{1+v^2} \\ K\{x\} + \frac{1}{v}K\{y\} - y(0) &= 0 \end{aligned} \right\} \quad (22)$$

Using (15) in (22), we have

$$\left. \begin{aligned} \frac{1}{v}K\{x\} + K\{y\} &= \frac{2v}{1+v^2} \\ K\{x\} + \frac{1}{v}K\{y\} &= 1 \end{aligned} \right\} \quad (23)$$

Solving the system (23) for $K\{x\}$ and $K\{y\}$, we have

$$\left. \begin{aligned} K\{x\} &= \left[\frac{v^2}{1+v^2} \right] \\ K\{y\} &= \left[\frac{v}{1+v^2} \right] \end{aligned} \right\} \quad (24)$$

Now taking inverse Kamal transform of system (24), we have

$$\left. \begin{aligned} x &= \sin t \\ y &= \cos t \end{aligned} \right\} \quad (25)$$

which is the required solution of (14) with (15).

8.3 Consider a system of linear ordinary differential equations

$$\left. \begin{aligned} \frac{dz}{dt} + x &= \sin t \\ \frac{dx}{dt} - y &= e^t \\ \frac{dy}{dt} + z + x &= 1 \end{aligned} \right\} \quad (26)$$

$$\text{with } x(0) = 1, y(0) = 1, z(0) = 0 \quad (27)$$

Solution using Mohand transforms:

Taking Mohand transform of system (26), we have

$$\left. \begin{aligned} M\left\{\frac{dz}{dt}\right\} + M\{x\} &= M\{\sin t\} \\ M\left\{\frac{dx}{dt}\right\} - M\{y\} &= M\{e^t\} \\ M\left\{\frac{dy}{dt}\right\} + M\{z\} + M\{x\} &= M\{1\} \end{aligned} \right\} \quad (28)$$

Now using the property, Mohand transform of the derivatives of the function, in (28), we have

$$\left. \begin{aligned} vM\{z\} - v^2z(0) + M\{x\} &= \left[\frac{v^2}{(v^2+1)} \right] \\ vM\{x\} - v^2x(0) - M\{y\} &= \left[\frac{v^2}{v-1} \right] \\ vM\{y\} - v^2y(0) + M\{z\} + M\{x\} &= v \end{aligned} \right\} \quad (29)$$

Using (27) in (29), we have

$$\left. \begin{aligned} vM\{z\} + M\{x\} &= \left[\frac{v^2}{(v^2+1)} \right] \\ vM\{x\} - M\{y\} &= \left[\frac{v^3}{v-1} \right] \\ vM\{y\} + M\{z\} + M\{x\} &= v + v^2 \end{aligned} \right\} \quad (30)$$

Solving the system (30) for $M\{x\}$, $M\{y\}$ and $M\{z\}$, we have

$$\left. \begin{aligned} M\{x\} &= \left[\frac{v^2}{v-1} \right] + \left[\frac{v^2}{(v^2+1)} \right] \\ M\{y\} &= \left[\frac{v^3}{(v^2+1)} \right] \\ M\{z\} &= v - \left[\frac{v^2}{v-1} \right] \end{aligned} \right\} \quad (31)$$

Now taking inverse Mohand transform of system (31), we have

$$\left. \begin{aligned} x &= e^t + \sin t \\ y &= \cos t \\ z &= 1 - e^t \end{aligned} \right\} \quad (32)$$

which is the required solution of (26) with (27).

Solution using Kamal transforms:

Taking Kamal transform of system (26), we have

$$\left. \begin{aligned} K\left\{\frac{dz}{dt}\right\} + K\{x\} &= K\{\sin t\} \\ K\left\{\frac{dx}{dt}\right\} - K\{y\} &= K\{e^t\} \\ K\left\{\frac{dy}{dt}\right\} + K\{z\} + K\{x\} &= K\{1\} \end{aligned} \right\} \quad (33)$$

Now using the property, Kamal transform of the derivatives of the function, in (33), we have

$$\left. \begin{aligned} \frac{1}{v}K\{z\} - z(0) + K\{x\} &= \left[\frac{v^2}{1+v^2} \right] \\ \frac{1}{v}K\{x\} - x(0) - K\{y\} &= \left[\frac{v}{1-v} \right] \\ \frac{1}{v}K\{y\} - y(0) + K\{z\} + K\{x\} &= v \end{aligned} \right\} \quad (34)$$

Using (27) in (34), we have

$$\left. \begin{aligned} \frac{1}{v}K\{z\} + K\{x\} &= \left[\frac{v^2}{(v^2+1)} \right] \\ \frac{1}{v}K\{x\} - K\{y\} &= \left[\frac{1}{1-v} \right] \\ \frac{1}{v}K\{y\} + K\{z\} + K\{x\} &= v + 1 \end{aligned} \right\} \quad (35)$$

Solving the system (35) for $K\{x\}$, $K\{y\}$ and $K\{z\}$, we have

$$\left. \begin{aligned} K\{x\} &= \left[\frac{v}{1-v} \right] + \left[\frac{v^2}{1+v^2} \right] \\ K\{y\} &= \left[\frac{v}{1+v^2} \right] \\ K\{z\} &= v - \left[\frac{v}{1-v} \right] \end{aligned} \right\} \quad (36)$$

Now taking inverse Kamal transform of system (36), we have

$$\left. \begin{aligned} x &= e^t + \sin t \\ y &= \cos t \\ z &= 1 - e^t \end{aligned} \right\} \quad (37)$$

which is the required solution of (26) with (27).

IX. CONCLUSIONS

In this paper, we have successfully discussed the comparative study of Mohand and Kamal transforms. In application section, we solve systems of differential equations (Homogeneous & Non-Homogeneous) comparatively using both the transforms. The numerical applications which are given in application section show that both the transforms (Mohand and Kamal transforms) are closely connected to each other.

REFERENCES

1. Lokenath Debnath and Bhatta, D., *Integral transforms and their applications*, Second edition, Chapman & Hall/CRC, 2006.
2. Aboodh, K.S., *The new integral transform "Aboodh Transform"*, *Global Journal of Pure and Applied Mathematics*, 9(1), 35-43, 2013.
3. Abdelilah, K. and Hassan, S., *The new integral transform "Kamal Transform"*, *Advances in Theoretical and Applied Mathematics*, 11(4), 451-458, 2016.
4. Elzaki, T.M., *The new integral transform "Elzaki Transform"*, *Global Journal of Pure and Applied Mathematics*, 1, 57-64, 2011.
5. Mahgoub, M.A.M., *The new integral transform "Mahgoub Transform"*, *Advances in Theoretical and Applied Mathematics*, 11(4), 391-398, 2016.
6. Mohand, M. and Mahgoub, A., *The new integral transform "Mohand Transform"*, *Advances in Theoretical and Applied Mathematics*, 12(2), 113 – 120, 2017.
7. Raisinghania, M.D., *Advanced differential equations*, S. Chand & Co. Ltd, 2015.
8. Jeffrey, A., *Advanced engineering mathematics*, Harcourt Academic Press, 2002.
9. Stroud, K.A. and Booth, D.J., *Engineering mathematics*, Industrial Press, Inc., 2001.
10. Greenberg, M.D., *Advanced engineering mathematics*, Prentice Hall, 1998.
11. Dass, H.K., *Advanced engineering mathematics*, S. Chand & Co. Ltd, 2007.
12. Watugula, G.K., *Sumudu transform: A new integral transform to solve differential equations and control engineering problems*, *International Journal of Mathematical Education in Science and Technology*, 24(1), 35-43, 1993.
13. Aggarwal, S., Sharma, N., Chauhan, R., Gupta, A.R. and Khandelwal, A., *A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients*, *Journal of Computer and Mathematical Sciences*, 9(6), 520-525, 2018.
14. Aggarwal, S., Chauhan, R. and Sharma, N., *A new application of Mahgoub transform for solving linear Volterra integral equations*, *Asian Resonance*, 7(2), 46-48, 2018.
15. Aggarwal, S., Sharma, N. and Chauhan, R., *Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform*, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(5), 173-176, 2018.
16. Aggarwal, S., Sharma, N. and Chauhan, R., *Application of Mahgoub transform for solving linear Volterra integral equations of first kind*, *Global Journal of Engineering Science and Researches*, 5(9), 154-161, 2018.
17. Aggarwal, S., Pandey, M., Asthana, N., Singh, D.P. and Kumar, A., *Application of Mahgoub transform for solving population growth and decay problems*, *Journal of Computer and Mathematical Sciences*, 9(10), 1490-1496, 2018.
18. Elzaki, T.M. and Ezaki, S.M., *On the Elzaki transform and ordinary differential equation with variable coefficients*, *Advances in Theoretical and Applied Mathematics*, 6(1), 41-46, 2011.
19. Elzaki, T.M. and Ezaki, S.M., *Applications of new transform "Elzaki transform" to partial differential equations*, *Global Journal of Pure and Applied Mathematics*, 7(1), 65-70, 2011.
20. Shendkar, A.M. and Jadhav, P.V., *Elzaki transform: A solution of differential equations*, *International Journal of Science, Engineering and Technology Research*, 4(4), 1006-1008, 2015.
21. Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R., *Application of Elzaki transform for solving population growth and decay problems*, *Journal of Emerging Technologies and Innovative Research*, 5(9), 281-284, 2018.
22. Aggarwal, S., Chauhan, R. and Sharma, N., *Application of Elzaki transform for solving linear Volterra integral equations of first kind*, *International Journal of Research in Advent Technology*, 6(12), 3687-3692, 2018.
23. Aboodh, K.S., *Application of new transform "Aboodh Transform" to partial differential equations*, *Global Journal of Pure and Applied Mathematics*, 10(2), 249-254, 2014.
24. Aboodh, K.S., Farah, R.A., Almardy, I.A. and Osman, A.K., *Solving delay differential equations by Aboodh transformation method*, *International Journal of Applied Mathematics & Statistical Sciences*, 7(2), 55-64, 2018.
25. Aboodh, K.S., Farah, R.A., Almardy, I.A. and Almostafa, F.A., *Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods*, *Global Journal of Pure and Applied Mathematics*, 13(8), 4347-4360, 2016.

26. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind, *International Journal of Research in Advent Technology*, 6(6), 1186-1190, 2018.
27. Aggarwal, S., Sharma, N. and Chauhan, R., A new application of Aboodh transform for solving linear Volterra integral equations, *Asian Resonance*, 7(3), 156-158, 2018.
28. Mohand, D., Aboodh, K.S. and Abdelbagy, A., On the solution of ordinary differential equation with variable coefficients using Aboodh transform, *Advances in Theoretical and Applied Mathematics*, 11(4), 383-389, 2016.
29. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of population growth and decay problems by using Mohand transform, *International Journal of Research in Advent Technology*, 6(11), 3277-3282, 2018.
30. Aggarwal, S., Chauhan, R. and Sharma, N., Mohand transform of Bessel's functions, *International Journal of Research in Advent Technology*, 6(11), 3034-3038, 2018.
31. Kumar, P.S., Saranya, C., Gnanavel, M.G. and Viswanathan, A., Applications of Mohand transform for solving linear Volterra integral equations of first kind, *International Journal of Research in Advent Technology*, 6(10), 2786-2789, 2018.
32. Kumar, P.S., Gomathi, P., Gowri, S. and Viswanathan, A., Applications of Mohand transform to mechanics and electrical circuit problems, *International Journal of Research in Advent Technology*, 6(10), 2838-2840, 2018.
33. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integral equations of second kind using Mohand transform, *International Journal of Research in Advent Technology*, 6(11), 3098-3102, 2018.
34. Sathya, S. and Rajeswari, I., Applications of Mohand transform for solving linear partial integro-differential equations, *International Journal of Research in Advent Technology*, 6(10), 2841-2843, 2018.
35. Kumar, P.S., Gnanavel, M.G. and Viswanathan, A., Application of Mohand transform for solving linear Volterra integro-differential equations, *International Journal of Research in Advent Technology*, 6(10), 2554-2556, 2018.
36. Aggarwal, S., Kamal transform of Bessel's functions, *International Journal of Research and Innovation in Applied Science*, 3(7), 1-4, 2018.
37. Abdelilah, K. and Hassan, S., The use of Kamal transform for solving partial differential equations, *Advances in Theoretical and Applied Mathematics*, 12(1), 7-13, 2017.
38. Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Kamal transform for solving linear Volterra integral equations, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(4), 138-140, 2018.
39. Gupta, A.R., Aggarwal, S. and Agrawal, D., Solution of linear partial integro-differential equations using Kamal transform, *International Journal of Latest Technology in Engineering, Management & Applied Science*, 7(7), 88-91, 2018.
40. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Kamal transform for solving linear Volterra integral equations of first kind, *International Journal of Research in Advent Technology*, 6(8), 2081-2088, 2018.
41. Aggarwal, S., Gupta, A.R., Asthana, N. and Singh, D.P., Application of Kamal transform for solving population growth and decay problems, *Global Journal of Engineering Science and Researches*, 5(9), 254-260, 2018.
42. Aggarwal, S. and Chaudhary, R., A comparative study of Mohand and Laplace transforms, *Journal of Emerging Technologies and Innovative Research*, 6(2), 230-240, 2019.